Photon blockade induced by atoms with Rydberg coupling

Jin-Feng Huang,[1] Jie-Qiao Liao,[2,3] and C. P. Sun[1,*]
[1] *State Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Science, Beijing 100190, China*
[2] *Department of Physics and Institute of Theoretical Physics, The Chinese University of Hong Kong, Shatin, Hong Kong SAR, China*
[3] *Advanced Science Institute, RIKEN, Wako-shi, Saitama 351-0918, Japan*
(Dated: September 19, 2012)

We study the photon blockade of the correlated two-photon scattering in a one-dimensional waveguide, which couples to two atoms with the Rydberg interaction. We obtain the analytic solution to the scattering problem of photonic wave packets on the two-coupled atoms. We examine the photon correlation by addressing the two-photon relative wave function and the second-order correlation function in the single- and two-photon resonance cases. It is found that, in the single-photon resonance condition, photon bunching and antibunching exist respectively in the transmitted and reflected two photons. In particular, the induced bunching and antibunching become stronger with the increase of the inter-atom coupling strength. In addition, we find a phenomenon of bunching-antibunching transition caused by the two-photon resonance.

PACS numbers: 42.50.Ar, 42.50.Pq, 42.79.Gn, 42.65..k

I. INTRODUCTION

Controllable transport of photons in one dimension (1D) waveguides is crucial for realizing all-optical quantum devices, which are basic elements for implementation of photonic quantum information processing. As an example, single-photon transistor [1], controlling the propagation of signal photons by another single photon, can be used to implement a single-photon quantum switch. Up to now, there have been some proposals for coherent transport of single- and two-photons in waveguides with linear and nonlinear dispersion relations [2–18]. Most of them are based on the physical mechanism of photon scattering by tunable targets.

Two-photon transport in a waveguide is a nontrivial mission since nonlinear interaction between photons [19, 20] may be induced by nonlinear scattering targets such as a nonlinear cavity [12] and strongly interacting atoms [21]. In particular, there exists a kind of nonlinear interaction could lead to photon blockade [19, 20], which can be used to realize a single-photon source. Because the photon nonlinearity in a cavity with atoms is a second-order effect of the interactions between cavity field and atoms [19], it is quite natural to ask how the atoms could mediate such a nonlinearity for photon blockade. Recently, Shi *et al.* [22] have considered two-photon scattering in a waveguide coupled to a cavity containing a two-level atom. Their results on photon blockade can well fit the experiment data [20].

We note that the blockade effect can also occur among atoms. In atomic blockade, double excitation of atoms is strongly suppressed by some inter-atom coupling [23–28]. Physically, when the interaction between excited atoms is strong enough, it will be difficult to simultaneously excite two atoms due to the coupling induced elevation of total energy. Since the simultaneous excitation of two atoms needs to absorb two photons, the atomic blockade will suppress the two-photon absorption one after another. In other words, the atomic blockade can induce the photon blockade under certain conditions. With this motivation, we will investigate in this paper how the atomic blockade [24, 28] exert an influence on the photon blockade [19, 20].

Deeply understanding the relation between these two blockade effects (atomic blockade and photon blockade) will provide a straight forward way to probe the nature of the inter-atom coupling through photon blockade. It is also possible to display the atomic blockade effect due to the photon blockade phenomenon. To illustrate the physical mechanism behind these scientific and technical issues, we study a hybrid system composed by a 1D linear waveguide, which contains two inter-coupled two-level Rydberg atoms. To understand the single photon contributions in the two photon process, we first calculate spatial evolution of the single photon wave packet in 1D waveguide. The similar approach has been used in Refs. [10, 12] to study the spatial wave packet scattering problem. Then we calculate the evolution of the two-photon spatial wave packet in a 1D waveguide. Since the second-order correlation function $g^{(2)}$ can describe photon blockade effect, we calculate $g^{(2)}$ for both the reflected and transmitted photons respectively. We find the second-order correlation function $g^{(2)}$ is proportional to the joint probability of the photons with some distance. We can predict the photon statistical properties by the wave function of the two photons. Our calculation shows the spatial distribution of the two photons is strongly dependent on the inter-atom coupling. The transmitted photons are strongly bunched except for some special inter-coupling values, which depends on the initial

* suncp@itp.ac.cn; http://power.itp.ac.cn/ suncp/index.html

conditions. The reflected photons are strongly repulse each other, namely the photon blockade effect, when the value of inter-atom coupling is away from the resonance, which means the inter-atom coupling equals the sum of each photon's center detuning. On the contrary, at this resonance, the reflected two photons exhibits bunching behavior. We can use this sole bunching behavior for the reflected two photons to probe the nature of the inter-atom coupling.

This paper is organized as follows. In Sec. II, we introduce the physical model and its Hamiltonian. In Secs. III and IV, we consider the single- and two-photon scattering with the Laplace transformation, respectively. In particular, we describe the two-photon correlation in the two-photon relative coordinate space, and calculate the second-order correlation function in two-photon reflection and transmission. Finally, we draw our conclusion in Sec. V and present the detailed derivation of two-photon solution in Appendix.

II. MODEL SETUP

We start by considering a 1D linear waveguide, which contains two two-level atoms coupled via the Rydberg interaction (see Fig. 1). The model Hamiltonian (with $\hbar=1$) of the system reads as

$$\begin{aligned}\hat{H}=&\int_0^\infty dk\omega_k(\hat{r}_k^\dagger\hat{r}_k+\hat{l}_k^\dagger\hat{l}_k)+\frac{\omega_0}{2}(\hat{\sigma}_1^z+\hat{\sigma}_2^z)\\&+g_0\int_0^\infty dk\sum_{\ell=1,2}[\hat{\sigma}_\ell^+(\hat{r}_k+\hat{l}_k)+(\hat{r}_k^\dagger+\hat{l}_k^\dagger)\hat{\sigma}_\ell^-]\\&+\xi|e\rangle_1\langle e|_1\otimes|e\rangle_2\langle e|_2.\end{aligned}\tag{1}$$

The first line of Eq. (1) is the free Hamiltonian of the fields and atoms. The creation (annihilation) operators $\hat{r}_k^\dagger$ ($\hat{r}_k$) and $\hat{l}_k^\dagger$ ($\hat{l}_k$) describe, respectively, the right- and left-propagating fields in the waveguide, with wave number k and frequency $\omega_k=v_pk$ (hereafter we take light velocity $v_p=1$). Pauli's operators $\hat{\sigma}_\ell^{x,y,z}$ $[\hat{\sigma}_\ell^\pm=\frac{1}{2}(\hat{\sigma}_\ell^x\pm i\hat{\sigma}_\ell^y)]$ are introduced to represent the ℓth ($\ell=1,2$) atom with energy level spacing ω_0. The Hamiltonian in the second-line of Eq. (1) depicts the atom-field interaction with the coupling strength g_0. In addition, the last term in Eq. (1) stands for the Rydberg interaction of strength ξ between the two atoms [29, 30].

By introducing the even- and odd-parity modes

$$\hat{b}_k=\frac{1}{\sqrt{2}}(\hat{r}_k+\hat{l}_k),\qquad \hat{c}_k=\frac{1}{\sqrt{2}}(\hat{r}_k-\hat{l}_k),\tag{2}$$

the Hamiltonian (1) can be decomposed into two parts, $\hat{H}=\hat{H}^{(o)}+\hat{H}^{(e)}$ with $\hat{H}^{(o)}=\int_0^\infty dk\omega_k\hat{c}_k^\dagger\hat{c}_k$ and

$$\begin{aligned}\hat{H}^{(e)}=&\int_0^\infty dk\omega_k\hat{b}_k^\dagger\hat{b}_k+\frac{\omega_0}{2}(\hat{\sigma}_1^z+\hat{\sigma}_2^z)\\&+g\int_0^\infty dk[(\hat{\sigma}_1^++\hat{\sigma}_2^+)\hat{b}_k+\hat{b}_k^\dagger(\hat{\sigma}_1^-+\hat{\sigma}_2^-)]\\&+\xi|e\rangle_1\langle e|_1\otimes|e\rangle_2\langle e|_2,\end{aligned}\tag{3}$$

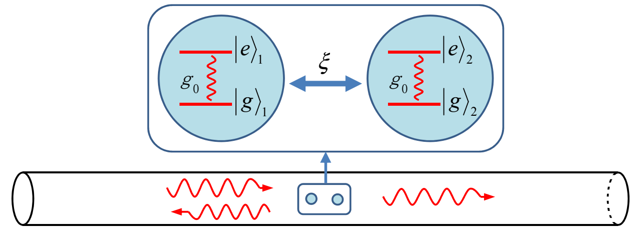

FIG. 1. Schematic diagram of the physical setup. Two interacting atoms are placed in a 1D linear waveguide. Photons injected from the left-hand side of the waveguide are scattered by the atoms.

where $g=\sqrt{2}g_0$. Obviously, the odd-parity modes decouple with the atoms so that their evolution is free. In the following we will mainly deal with the evolution of the even-parity modes.

In a rotating frame with respect to

$$\hat{H}_0^{(e)}=\omega_0\int_0^\infty dk\hat{b}_k^\dagger\hat{b}_k+\frac{\omega_0}{2}(\hat{\sigma}_1^z+\hat{\sigma}_2^z),\tag{4}$$

the Hamiltonian $\hat{H}^{(e)}$ becomes

$$\begin{aligned}\hat{H}_I^{(e)}=&\int_0^\infty dk\Delta_k\hat{b}_k^\dagger\hat{b}_k+\xi|e\rangle_1\langle e|_1\otimes|e\rangle_2\langle e|_2\\&+g\int_0^\infty dk[(\hat{\sigma}_1^++\hat{\sigma}_2^+)\hat{b}_k+\hat{b}_k^\dagger(\hat{\sigma}_1^-+\hat{\sigma}_2^-)],\end{aligned}\tag{5}$$

where $\Delta_k=\omega_k-\omega_0$. Based on Hamiltonian (5), we can work out the photon scattering solution for the system. To comprehensively understand the physical picture for the photon scattering, we will firstly consider the single-photon scattering, and then we will study how does the Rydberg interaction affect the two-photon scattering.

III. SINGLE-PHOTON SCATTERING

In the following discussion, we employ the time-dependent approach to the wave packet scattering [10, 12] other than the approach based on stationary state [2–5, 9, 13, 15] to study our photon scattering problem. We note that the total excitation number operator $\hat{N}=\hat{a}^\dagger\hat{a}+|e\rangle_1\langle e|_1+|e\rangle_2\langle e|_2$ in this system is a conserved quantity due to $[\hat{N},\hat{H}_I^{(e)}]=0$. Consequently, the Hilbert space of the system can be divided into the direct-sum subspaces with different excitations. For single-photon scattering, it is sufficient to consider the scattering problems within the single-excitation subspace. In this subspace, an arbitrary state of the system can be expressed as

$$\begin{aligned}|\Phi(t)\rangle=&\alpha_1(t)|\emptyset\rangle|e\rangle_1|g\rangle_2+\alpha_2(t)|\emptyset\rangle|g\rangle_1|e\rangle_2\\&+\int_0^\infty dk\beta_k(t)\hat{b}_k^\dagger|\emptyset\rangle|g\rangle_1|g\rangle_2,\end{aligned}\tag{6}$$

where $|\emptyset\rangle$ is the vacuum state and $|1_k\rangle$ represents the state with a single photon in the kth even mode of

the waveguide. The variables $\alpha_1(t)$, $\alpha_2(t)$, and $\beta_k(t)$ are probability amplitudes. In terms of the Schrödinger equation $i\,(\partial/\partial t)\,|\Phi(t)\rangle = \hat{H}_I^{(e)}|\Phi(t)\rangle$, we obtain the following equation of motion for probability amplitudes

$$\begin{aligned}
\dot{\alpha}_1(t) &= -ig\int_0^\infty dk\beta_k(t),\\
\dot{\alpha}_2(t) &= -ig\int_0^\infty dk\beta_k(t),\\
\dot{\beta}_k(t) &= -i\Delta_k\beta_k(t) - ig[\alpha_1(t)+\alpha_2(t)].
\end{aligned} \tag{7}$$

For single-photon scattering, we assume that the two atoms are initially in ground state and a single even-mode photon is in a Lorentzian wave packet. The initial condition reads

$$\begin{aligned}
&\alpha_1(0) = 0, \qquad \alpha_2(0) = 0,\\
&\beta_k(0) = G_1\frac{e^{i\Delta_k l}}{\Delta_k - \delta + i\epsilon}, \qquad l \geq 0,
\end{aligned} \tag{8}$$

where $G_1 = \sqrt{\epsilon/\pi}$ is the normalization constant, and l is the initial distance between the wavefront of the photon wave packet and the atoms. In addition, δ and ϵ are, respectively, the center detuning and width of the wave packet. The transient solution of these probability amplitudes might be obtained by the Laplace transformation method. For the motivation of scattering problem, we focus on the long-time solution,

$$\begin{aligned}
&\alpha_1(t\to\infty) = 0, \qquad \alpha_2(t\to\infty) = 0,\\
&\beta_k(t\to\infty) = \bar{t}_k\beta_k(0)e^{-i\Delta_k t},
\end{aligned} \tag{9}$$

where we introduce the phase factor

$$\bar{t}_k = \frac{\Delta_k - i\gamma}{\Delta_k + i\gamma} \tag{10}$$

with $\gamma = 2\pi g^2$ being the decay rate of the atom.

In practice, a single photon should be injected in right- or left-propagating modes. Hence, we need to consider the single-photon scatting in modes $\hat{r}_k$ and $\hat{l}_k$. For the right-propagating injection, the initial state is

$$\begin{aligned}
|\psi(0)\rangle &= \int_0^\infty dk\beta_k(0)\hat{r}_k^\dagger|\emptyset\rangle\\
&= \frac{1}{\sqrt{2}}\int_0^\infty dk\beta_k(0)(\hat{b}_k^\dagger + \hat{c}_k^\dagger)|\emptyset\rangle.
\end{aligned} \tag{11}$$

The wave function in position space reads

$$\langle x|\psi(0)\rangle \simeq -i\sqrt{2\pi}G_1 e^{i(\delta-i\epsilon)(x+l)}\theta(-x-l), \tag{12}$$

where $\theta(x)$ is the Heaviside step function. In the long-time limit, the single-photon wave function is

$$|\psi(t\to\infty)\rangle = \int_0^\infty dk\beta_k(0)e^{-i\Delta_k t}(T_k\hat{r}_k^\dagger + R_k\hat{l}_k^\dagger)|\emptyset\rangle, \tag{13}$$

where the transmission and reflection amplitudes are obtained as

$$T_k = \frac{\Delta_k}{\Delta_k + i\gamma}, \qquad R_k = \frac{-i\gamma}{\Delta_k + i\gamma}. \tag{14}$$

The output wave function in position space is

$$\langle x|\psi(t\to\infty)\rangle = \psi_r(x,l,\delta) + \psi_l(x,l,\delta), \tag{15}$$

where

$$\begin{aligned}
\psi_r(x,l,\delta) &= -i\sqrt{2\pi}G_1 T_{\delta-i\epsilon}e^{i(\delta-i\epsilon)(x-t+l)}\theta(-x+t-l),\\
\psi_l(x,l,\delta) &= -i\sqrt{2\pi}G_1 R_{\delta-i\epsilon}e^{-i(\delta-i\epsilon)(x+t-l)}\theta(x+t-l).
\end{aligned} \tag{16}$$

We can see from Eq. (16) that the waveform (proportional to $|\psi_r(x,l,\delta)|^2$ and $|\psi_l(x,l,\delta)|^2$) of the transmitted and reflected photon is the same as that for the input state (12), whereas the amplitudes are normalized by the transmission and reflection coefficients ($|T_{\delta-i\epsilon}|^2$ and $|R_{\delta-i\epsilon}|^2$).

IV. TWO-PHOTON SCATTERING

We now turn to study the two-photon scattering. We will use the Laplace transformation to obtain the long-time state of two photons, and special attention will be paid to clarifying the relationship between the atom blockade and the photon blockade.

A. Equations of motion and solution

In the two-excitation subspace, there are four types of basis states: two excitations in atoms (with basis $|\emptyset\rangle|e\rangle_1|e\rangle_2$), one in atoms and the other in fields (with bases $|1_k\rangle|e\rangle_1|g\rangle_2$ and $|1_k\rangle|g\rangle_1|e\rangle_2$), two in fields (with basis $|1_p,1_q\rangle|g\rangle_1|g\rangle_2$). Then an arbitrary state in this subspace is written as

$$\begin{aligned}
|\varphi(t)\rangle = {}& A(t)|\emptyset\rangle|e\rangle_1|e\rangle_2 + \int_0^\infty dkB_k(t)|1_k\rangle|e\rangle_1|g\rangle_2\\
&+\int_0^\infty dkC_k(t)|1_k\rangle|g\rangle_1|e\rangle_2\\
&+\int_0^\infty dp\int_0^p dqD_{p,q}(t)|1_p,1_q\rangle|g\rangle_1|g\rangle_2,
\end{aligned} \tag{17}$$

where $A(t)$, $B_k(t)$, $C_k(t)$, and $D_{p,q}(t)$ are probability amplitudes. From the Schrödinger equation, we get the following equations of motion for probability amplitudes,

$$\begin{aligned}
\dot{A}(t) &= -i\xi A(t) - ig\int_0^\infty dk[B_k(t)+C_k(t)],\\
\dot{B}_k(t) &= -i\Delta_k B_k(t) - igA(t) - ig\int_0^\infty dpD_{p,k}(t),\\
\dot{C}_k(t) &= -i\Delta_k C_k(t) - igA(t) - ig\int_0^\infty dpD_{p,k}(t),\\
\dot{D}_{p,q}(t) &= -i(\Delta_p+\Delta_q)D_{p,q}(t) - ig[B_p(t)+B_q(t)]\\
&\quad -ig[C_p(t)+C_q(t)],
\end{aligned} \tag{18}$$

where we have used the symmetry relation $D_{k,p}(t) = D_{p,k}(t)$.

We consider the initial state where the two atoms are in their ground state and the two photons are in a wave packet. The corresponding initial condition reads

$$\begin{aligned} A(0) = 0, \qquad B_k(0) = 0, \qquad C_k(0) = 0, \\ D_{p,q}(0) = G_2 \left(\frac{e^{i\Delta_p l_1}}{\Delta_p - \delta_1 + i\epsilon} \frac{e^{i\Delta_q l_2}}{\Delta_q - \delta_2 + i\epsilon} \right. \\ \left. + \frac{e^{i\Delta_q l_1}}{\Delta_q - \delta_1 + i\epsilon} \frac{e^{i\Delta_p l_2}}{\Delta_p - \delta_2 + i\epsilon} \right), \end{aligned} \tag{19}$$

where l_ℓ is the initial position of the ℓth ($\ell = 1, 2$) photon. Without loss of generality, we assume $l_1 \geq 0$, $l_2 \geq 0$, and $l_1 \geq l_2$ below. The normalization constant reads

$$G_2 = \frac{\epsilon}{\pi} \left[1 + \frac{4\epsilon^2 e^{-2\epsilon(l_1 - l_2)}}{(\delta_1 - \delta_2)^2 + 4\epsilon^2} \right]^{-1/2}, \tag{20}$$

Under the above initial condition, the long-time solution for these probability amplitudes might be obtained with the Laplace transformation, which satisfy $A(t \to \infty) = 0$, $B_k(t \to \infty) = 0$, $C_k(t \to \infty) = 0$, and

$$D_{p,q}(t \to \infty) = (\bar{t}_p \bar{t}_q D_{p,q}(0) + J_{p,q}) e^{-i(\Delta_p + \Delta_q)t}, \tag{21}$$

where $\bar{t}_p$ ($\bar{t}_q$) has been defined by Eq. (10) and

$$\begin{aligned} J_{p,q} = 4G_2\gamma^2 \frac{e^{i(\Delta_p+\Delta_q)l_1}}{(\Delta_p + i\gamma)(\Delta_q + i\gamma)} \frac{(\Delta_p + \Delta_q - 2\xi)}{(\Delta_p + \Delta_q - \xi + i\gamma)} \\ \times \frac{1}{(\Delta_p + \Delta_q - \delta_1 - \delta_2 + 2i\epsilon)} \\ \times \left[\left(\frac{e^{-(i\delta_2+\epsilon)(l_1-l_2)}}{i\gamma + \delta_2 - i\epsilon} + \frac{e^{-(i\delta_2+\epsilon)(l_1-l_2)}}{\Delta_p + \Delta_q - \delta_2 + i\epsilon + i\gamma} \right) \right. \\ \left. - \left(\frac{e^{-\gamma(l_1-l_2)}}{i\gamma + \delta_2 - i\epsilon} - \frac{e^{-\gamma(l_1-l_2)}}{\Delta_p + \Delta_q - \delta_1 + i\epsilon + i\gamma} \right) \right]. \end{aligned} \tag{22}$$

The first term in Eq. (21) describes two-photon independent scattering process, while the second term represents photon correlation induced by scattering. It should be pointed out that, when $\xi = 0$, $J_{p,q} \neq 0$. This fact means that the photon correlation can be induced even in the absence of the Rydberg interaction.

B. Two-photon wave functions in real space

We consider a realistic case with the two photons injected from the left-hand side of the waveguide. Then, the initial state of the photons can be written as

$$|\psi(0)\rangle = \int_0^\infty dp \int_0^p dq D_{p,q}(0) \hat{r}_p^\dagger \hat{r}_q^\dagger |\emptyset\rangle. \tag{23}$$

In terms of the basis wave function

$$\langle x_1, x_2 | \hat{r}_p^\dagger \hat{r}_q^\dagger |0\rangle = \mathcal{N}_{rr} \left(e^{i\Delta_p x_1} e^{i\Delta_q x_2} + x_1 \leftrightarrow x_2 \right) \tag{24}$$

with $\mathcal{N}_{rr} = 1/(2\sqrt{2}\pi)$, the wave function in position space of the initial state can be written as

$$\begin{aligned} \langle x_1, x_2 | \psi(0) \rangle = -4\pi^2 \mathcal{N}_{rr} G_2 e^{(i\delta_1+\epsilon)(x_1+l_1)} e^{(i\delta_2+\epsilon)(x_2+l_2)} \\ \times \theta(-x_1 - l_1)\theta(-x_2 - l_2) + (x_1 \leftrightarrow x_2). \end{aligned} \tag{25}$$

By introducing the center-of-mass coordinate $x_c = (x_1+x_2)/2$, relative coordinate $x = x_1 - x_2$, total momentum $E = \delta_1 + \delta_2$ and relative momentum $\delta = (\delta_1 - \delta_2)/2$, the wave function (25) becomes

$$\begin{aligned} \langle x_1, x_2 | \psi(0) \rangle = -4\pi^2 \mathcal{N}_{rr} G_2 \left[e^{i\delta_1(x_c + x/2 + l_1)} e^{\epsilon(x_c + x/2 + l_1)} \right. \\ \times e^{i\delta_2(x_c - x/2 + l_2)} e^{\epsilon(x_c - x/2 + l_2)} \\ \times \theta(-x_c + x/2 - l_2)\theta(-x_c - x/2 - l_1) \\ \left. + (x \leftrightarrow -x) \right]. \end{aligned} \tag{26}$$

For the special case of $l_2 = l_1$, the wave function reduces to

$$\begin{aligned} \langle x_1, x_2 | \psi(0) \rangle = -8\pi^2 \mathcal{N}_{rr} G_2 e^{(iE+2\epsilon)(x_c+l_1)} \\ \times \cos(\delta x)\theta(-x_c - l_1 - |x|/2). \end{aligned} \tag{27}$$

In the long-time limit, the two-photon state can be expressed as

$$|\psi(t \to \infty)\rangle = |\psi_{rr}\rangle + |\psi_{rl}\rangle + |\psi_{lr}\rangle + |\psi_{ll}\rangle, \tag{28}$$

where

$$\begin{aligned} |\psi_{rr}\rangle &= \int_0^\infty \int_0^\infty dp dq D_{p,q}^{rr} e^{-i(\Delta_p+\Delta_q)t} \hat{r}_p^\dagger \hat{r}_q^\dagger |\emptyset\rangle, \\ |\psi_{rl}\rangle &= \int_0^\infty \int_0^\infty dp dq D_{p,q}^{rl} e^{-i(\Delta_p+\Delta_q)t} \hat{r}_p^\dagger \hat{l}_q^\dagger |\emptyset\rangle, \\ |\psi_{lr}\rangle &= \int_0^\infty \int_0^\infty dp dq D_{p,q}^{lr} e^{-i(\Delta_p+\Delta_q)t} \hat{l}_p^\dagger \hat{r}_q^\dagger |\emptyset\rangle, \\ |\psi_{ll}\rangle &= \int_0^\infty \int_0^\infty dp dq D_{p,q}^{ll} e^{-i(\Delta_p+\Delta_q)t} \hat{l}_p^\dagger \hat{l}_q^\dagger |\emptyset\rangle, \end{aligned} \tag{29}$$

with

$$\begin{aligned} D_{p,q}^{rr} &= \frac{1}{2}(T_p T_q D_{p,q}(0) + J_{p,q}/4), \\ D_{p,q}^{rl} &= \frac{1}{2}(T_p R_q D_{p,q}(0) + J_{p,q}/4), \\ D_{p,q}^{lr} &= \frac{1}{2}(R_p T_q D_{p,q}(0) + J_{p,q}/4), \\ D_{p,q}^{ll} &= \frac{1}{2}(R_p R_q D_{p,q}(0) + J_{p,q}/4). \end{aligned} \tag{30}$$

Here $D_{p,q}^{rr}$ and $D_{p,q}^{ll}$ are, respectively, the two-photon transmission and reflection amplitudes, while $D_{p,q}^{rl}$ and $D_{p,q}^{lr}$ are the one-photon reflection and one-photon transmission amplitudes.

C. Two-photon correlation in position variables

To characterize the photon correlation, it is convenient to consider the two-photon transmission or reflection cases. For the two-photon transmission, the output state of the two right-going photons is

$$\langle x_1, x_2|\psi_{rr}\rangle = -4\pi^2\mathcal{N}_{rr}G_2 e^{i(E-2i\epsilon)(x_c-t)} \times e^{i(\frac{E}{2}-i\epsilon)(l_1+l_2)}e^{i\delta(l_1-l_2)}\Phi_{rr}(x) \quad (31)$$

with

$$\begin{aligned}\Phi_{rr}(x) = & T_{\delta_1-i\epsilon}T_{\delta_2-i\epsilon}[e^{i\delta x}\theta(-x_c-x/2+t-l_1) \\ & \times\theta(-x_c+x/2+t-l_2) + x\leftrightarrow -x] \\ & -R_{\delta_1-i\epsilon}R_{\delta_2-i\epsilon}\frac{E-2i\epsilon-2\xi}{E-2i\epsilon-\xi+i\gamma} \\ & \times e^{i(\frac{E}{2}-i\epsilon+i\gamma)|x|}\theta(-x_c+t-|x|/2-l_1).\end{aligned} \quad (32)$$

When $l_2 = l_1$, the output state (31) reduces to

$$\langle x_1, x_2|\psi_{rr}\rangle = -8\pi^2\mathcal{N}_{rr}G_2 e^{(iE+2\epsilon)(x_c-t+l_1)} \times\theta(-x_c+t-l_1-|x|/2)\phi_{rr}(x), \quad (33)$$

where

$$\begin{aligned}\phi_{rr}(x) = & T_{\delta_1-i\epsilon}T_{\delta_2-i\epsilon}\cos(\delta x) \\ & -\frac{1}{2}R_{\delta_1-i\epsilon}R_{\delta_2-i\epsilon}\frac{E-2\xi-2i\epsilon}{E-\xi-2i\epsilon+i\gamma}e^{i(\frac{E}{2}-i\epsilon+i\gamma)|x|},\end{aligned} \quad (34)$$

which satisfies $\phi_{rr}(-x) = \phi_{rr}(x)$. In the derivation of Eq. (31), we have used the condition $\gamma \gg \epsilon$.

We note that Eq. (33) is a product of the center-of-mass wave function $\exp[(iE+2\epsilon)(x_c-t+l_1)]$ and the relative wave function $\phi_{rr}(x)$ in the region defined by the step function $\theta(-x_c+t-l_1-|x|/2)$. As $|\phi_{rr}(x)|^2$ is proportional to the joint probability for two photons with a separation x, it could be used to characterize the spatial statistics of the two photons. For example, the peak and dip feature around $|\phi_{rr}(0)|^2$, implies photon bunching and antibunching, respectively. In Eq. (34), the first term of $\phi_{rr}(x)$ describes an independent two-photon transmission process, while the second term is a two-photon correlation induced by scattering. Physically, the present system has two resonant scattering conditions: single- and two-photon resonances. When the frequency of the a single photon matches the energy separation of a single atom, i.e. $\delta_1 = \delta_2 = 0$ (or $E = 0$ and $\delta = 0$), the single photon will resonantly excite the atom. We call this as the single-photon resonance condition. On the other hand, when the total energy of the two photons equals to the energy required to excite the two coupled atoms, i.e., $\delta_1 + \delta_2 = \xi$ (or $E = \xi$), the two photons can resonantly excite the two coupled atoms, even the single-photon process could be off-resonant. This regime is called as two-photon resonance.

In the single-photon resonance case, the independent two-photon transmission process will be completely suppressed, and a pure photon-correlation effect can be seen from $|\phi_{rr}(x)|^2$. In Fig. 2(a), we plot $|\phi_{rr}(x)|^2$ as a function of ξ/γ and γx. When $\xi = 0$, there is no photon correlation [dash line in Fig. 2(b)]. This result corresponds to the fluorescence-complete-vanishing phenomenon found in Ref. [16]. For a nonzero ξ/γ, we can see clear evidence for photon bunching [Fig. 2(b)]. In particular, with the increasing of ξ/γ, $|\phi_{rr}(0)|^2$ increases gradually, and saturates when $\xi/\gamma \gg 1$. This means that the photon bunching becomes stronger for a larger ξ/γ in the single-photon resonance regime.

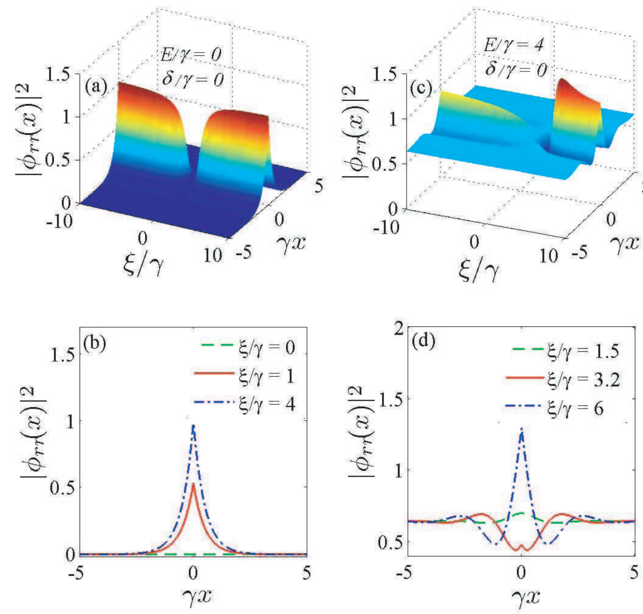

FIG. 2. (Color online) Plot of $|\phi_{rr}(x)|^2$ vs the scaled parameters ξ/γ and γx when (a) $E/\gamma = 0$, $\delta = 0$ and (b) $E/\gamma = 4$, $\delta = 0$. (c) and (d) are, respectively, the curves from (a) and (b) when the parameter ξ/γ takes several certain values. Here, we consider the near monochromatic limit $\epsilon/\gamma = 0.01$.

In the case with single-photon off resonance (e.g., $E/\gamma = 4$), we plot $|\phi_{rr}(x)|^2$ vs ξ/γ and γx in Figs. 2(c) and 2(d). The curves exhibit photon bunching in most region of ξ/γ, but there is also some oscillation pattern with respect to γx due to independent photon transmission. However, around the two-photon resonance, i.e., $\xi = E$, there is a clear evidence for the photon antibunching [Fig. 2(d)]. This interesting phenomenon of photon statistics is induced by the Rydberg coupling.

By the similar procedure, using the basis wave function

$$\langle x_1, x_2|\hat{l}_p^\dagger\hat{l}_q^\dagger|0\rangle = \mathcal{N}_{ll}\left(e^{-i\Delta_p x_1}e^{-i\Delta_q x_2} + x_1\leftrightarrow x_2\right) \quad (35)$$

with $\mathcal{N}_{ll} = 1/(2\sqrt{2}\pi)$, the output state of the two left-going photons in the long-time limit $t\to\infty$ is

$$\langle x_1, x_2|\psi_{ll}\rangle = -4\pi^2\mathcal{N}_{ll}G_2 e^{i(\frac{E}{2}-i\epsilon)(l_1+l_2)}e^{i\delta(l_1-l_2)} \times e^{i(E-2i\epsilon)(-x_c-t)}\Phi_{ll}(x) \quad (36)$$

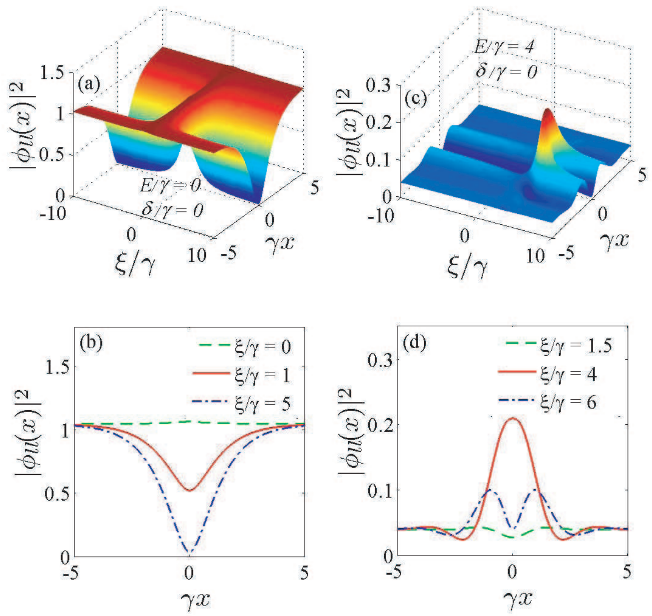

FIG. 3. (Color online) Plot of $|\phi_{ll}(x)|^2$ vs ξ/γ and γx when (a) $E/\gamma = 0$, $\delta = 0$ and (b) $E/\gamma = 4$, $\delta = 0$. (c) and (d) are, respectively, the curves from (a) and (b) when the parameter ξ/γ takes several certain values. Here, we take $\epsilon/\gamma = 0.01$.

with

$$\begin{aligned}\Phi_{ll}(x) \equiv\ & R_{\delta_1 - i\epsilon} R_{\delta_2 - i\epsilon}[e^{-i\delta x}\theta(x_c + x/2 + t - l_1) \\ & \times\theta(x_c - x/2 + t - l_2) + x \leftrightarrow -x] \\ & - R_{\delta_1 - i\epsilon} R_{\delta_2 - i\epsilon}\frac{E - 2i\epsilon - 2\xi}{E - 2i\epsilon - \xi + i\gamma} \\ & \times e^{i(\frac{E}{2} - i\epsilon + i\gamma)|x|}\theta(x_c + t - |x|/2 - l_1). \quad (37)\end{aligned}$$

When $l_1 = l_2$, the output state of two reflected photons becomes

$$\begin{aligned}\langle x_1, x_2|\psi_{ll}\rangle = & -8\pi^2 \mathcal{N}_{ll} G_2 e^{(iE+2\epsilon)(-x_c - t + l_1)} \\ & \times\theta(x_c + t - l_1 - |x|/2)\phi_{ll}(x) \quad (38)\end{aligned}$$

with

$$\begin{aligned}\phi_{ll}(x) = R_{\delta_1 - i\epsilon} R_{\delta_2 - i\epsilon}\Big[&\cos(\delta x) \\ & - \frac{1}{2}\frac{E - 2i\epsilon - 2\xi}{E - 2i\epsilon - \xi + i\gamma} e^{i(\frac{E}{2} - i\epsilon + i\gamma)|x|}\Big], \quad (39)\end{aligned}$$

In Figs. 3(a) and 3(b), we plot $|\phi_{ll}(x)|^2$ as a function of ξ/γ and γx in the single-photon resonance $E/\gamma = 0$ and $\delta/\gamma = 0$. Similar to the two-photon transmission, when $\xi = 0$, there is no photon correlation [dash line in Fig. 3(b)]. For nonzero ξ, we can see photon antibunching [Fig. 3(b)]. With the increasing of ξ/γ, the $|\phi_{ll}(0)|^2$ decreases gradually, and eventually approaches zero when $\xi/\gamma \gg 0$.

On the other hand, $|\phi_{ll}(x)|^2$ in the case with single-photon off resonance ($E/\gamma = 4$) is shown in Figs. 3(c) and 3(d). Though there exists some oscillation with respect to γx, we can still see photon antibunching in most region of the parameter ξ/γ. In addition, it is similar to the two-photon transmission in the sense that there also exists the photon statistics change induced by the two-photon resonance. Around $\xi = E$, we see clear evidence for photon bunching [Fig. 3(d)]. We find this bunching peak is exactly at $\xi = E$ for the reflected photons. This change of statistical properties can be used to detect the form of the interaction between atoms.

D. Second-order correlation function

We can also present a quantitative description of the statistics of the right- and left-going photons using the second-order correlation function $g^{(2)}$. In particular, we will only concern the two-photon reflection and transmission because the photon statistics in these two cases makes sense. From Eq. (28), the two-photon states are defined by

$$|\psi_{ss'}\rangle = \frac{1}{\sqrt{2}}\int\int dx_1 dx_2 \langle x_1, x_2|\psi_{ss'}\rangle \hat{\psi}_s^\dagger(x_1)\hat{\psi}_{s'}^\dagger(x_2)|\emptyset\rangle \quad (40)$$

for $s, s' = r, l$, where the field operators satisfy the bosonic commutation relation $[\hat{\psi}_s(x_1), \hat{\psi}_s^\dagger(x_2)] = \delta(x_1 - x_2)$. For state (40), the second-order correlation function is

$$g_s^{(2)}(\tau) = \frac{G_s^{(2)}(x_1, \tau)}{G_s^{(1)}(x_1)G_s^{(1)}(x_1 + \tau)}, \qquad s = r, l, \quad (41)$$

where

$$\begin{aligned}G_s^{(1)}(x) &= \langle\bar{\psi}_{ss}|\hat{\psi}_s^\dagger(x)\hat{\psi}_s(x)|\bar{\psi}_{ss}\rangle, \\ G_s^{(2)}(x_1, \tau) &= \langle\bar{\psi}_{ss}|\hat{\psi}_s^\dagger(x_1)\hat{\psi}_s^\dagger(x_1 + \tau)\hat{\psi}_s(x_1 + \tau)\hat{\psi}_s(x_1)|\bar{\psi}_{ss}\rangle, \quad (42)\end{aligned}$$

with $|\bar{\psi}_{ss}\rangle = |\psi_{ss}\rangle/\sqrt{\langle\psi_{ss}|\psi_{ss}\rangle}$. Then, by Eqs. (40) and (41), we get

$$g_s^{(2)}(\tau) = \frac{|\psi_{ss}(x_1, x_1 + \tau)|^2 \int\int dx_1 dx_2 |\psi_{ss}(x_1, x_2)|^2}{2\int dx_2 |\psi_{ss}(x_1, x_2)|^2 \int dx_2 |\psi_{ss}(x_1 + \tau, x_2)|^2}, \quad (43)$$

for $s = r, l$.

For the two transmitted photons, we introduce a new frame of reference as $x_1 = x_1' + t$ and $x_2 = x_2' + t$. Correspondingly, the center-of-mass and relative coordinates become $x_c' = (x_1' + x_2')/2$ and $x' = x_1' - x_2'$, which have the same form as those in the old frame of reference of x. In the new frame of reference, the state of the two right-going photons is

$$\begin{aligned}\langle x_1', x_2'|\psi_{rr}\rangle = & -8\pi^2 \mathcal{N}_{rr} G_2 e^{(iE+2\epsilon)(x_c' + l_1)} \\ & \times\theta(-x_c' - l_1 - |x'|/2)\phi_{rr}(x'), \quad (44)\end{aligned}$$

which is independent of the time t. In the following, we will restrict our calculation in the new frame of reference,

and omit the superscript "′" for simplicity. After some tedious calculations, we get

$$g_r^{(2)}(\tau) = F_r(x_1,\tau)|\phi_{rr}(\tau)|^2, \tag{45}$$

where

$$F_r(x_1,\tau) = M_r^{-1}\theta(-x_1-l_1-\tau)\theta(-x_1-l_1) \times \int_0^{+\infty} e^{-2\epsilon x}|\phi_{rr}(x)|^2 dx, \tag{46}$$

with

$$M_r = 4\epsilon\theta(-x_1-l_1-\tau)\theta(-x_1-l_1)e^{4\epsilon(x_1+l_1+\frac{\tau}{2})} \times \int_{x_1+l_1}^{\infty} dx e^{-2\epsilon x}|\phi_{rr}(x)|^2 \int_{x_1+\tau+l_1}^{\infty} dx e^{-2\epsilon x}|\phi_{rr}(x)|^2 \tag{47}$$

Similarly, for the two reflected photons, we also introduce a new frame of reference as $x_1 = x_1' - t$ and $x_2 = x_2' - t$, then the state for the two reflected photons becomes

$$\langle x_1', x_2'|\psi_{ll}\rangle = -8\pi^2\mathcal{N}_{ll}G_2 e^{(iE+2\epsilon)(-x_c'+l_1)} \times\theta(x_c'-l_1-|x'|/2)\phi_{ll}(x'), \tag{48}$$

and the correlation function $g_{ll}^{(2)}(\tau)$ reads

$$g_l^{(2)}(\tau) = F_l(x_1,\tau)|\phi_{ll}(\tau)|^2, \tag{49}$$

where

$$F_l(x_1,\tau) = M_l^{-1}\theta(x_1-l_1+\tau)\theta(x_1-l_1) \times \int_0^{+\infty} e^{-2\epsilon x}|\phi_{ll}(x)|^2 dx, \tag{50}$$

with

$$M_l = 4\epsilon\theta(x_1-l_1+\tau)\theta(x_1-l_1)e^{-4\epsilon(x_1-l_1+\frac{\tau}{2})} \times \int_{l_1-x_1}^{+\infty} e^{-2\epsilon x}|\phi_{ll}(x)|^2 dx \int_{l_1-(x_1+\tau)}^{+\infty} e^{-2\epsilon x}|\phi_{ll}(x)|^2 dx. \tag{51}$$

In Fig. 4, we illustrate the second-order correlation function $g_r^{(2)}(\tau)$ and $g_l^{(2)}(\tau)$ as a function of the scaled parameters ξ/γ and $\gamma\tau$. Here, the parameters are chosen the same as those in Figs. 2 and 3 for comparison. In the single-photon resonance, i.e., $E/\gamma = 0$ and $\delta/\gamma = 0$, we can see that, when $\xi/\gamma \neq 0$, $g_r^{(2)}(0) > g_r^{(2)}(\tau)$ [Fig. 4(a)] and $g_l^{(2)}(0) < g_l^{(2)}(\tau)$ [Fig. 4(c)], which represent photon bunching in transmission and photon antibunching in reflection, respectively. Interestingly, the $g_l^{(2)}(0)$ approaches zero when $\xi/\gamma \gg 1$, which means that the photon blockage phenomenon takes place in the reflection [Fig. 4(c)]. When $\xi/\gamma = 0$, the two photons are reflected completely and independently. At the point $\xi/\gamma = 0$ $g_l^{(2)}(\tau) = 1/2$ (for Fock state $|2\rangle$) and $g_r^{(2)}(\tau)$ does not make sense.

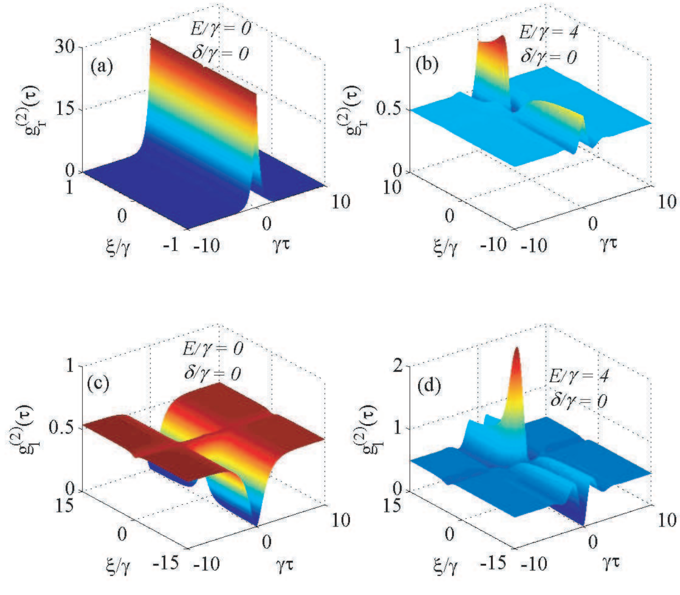

FIG. 4. (Color online) The second-order correlation function $g_r^{(2)}(\tau)$ and $g_l^{(2)}(\tau)$ vs ξ/γ and $\gamma\tau$ when (a,c) $E/\gamma = 0$, $\delta = 0$ and (b,d) $E/\gamma = 4$, $\delta = 0$.

On the other hand, Figs. 4(b) and 4(d) show that the two-photon resonance ($\xi = E$) will induce the photon statistics change between bunching and antibunching. For the two-photon transmission, we see $g_r^{(2)}(0) > g_r^{(2)}(\tau)$ (bunching) in most region of ξ/γ and $g_r^{(2)}(0) < g_r^{(2)}(\tau)$ (antibunching) around the two-photon resonance point $\xi = E$ [Fig. 4(b)]. For comparison, the two-photon reflection result is shown in Fig. 4(d). We can see $g_l^{(2)}(0) > g_l^{(2)}(\tau)$ (bunching) around $\xi = E$ and $g_l^{(2)}(0) < g_l^{(2)}(\tau)$ (antibunching) in other region. These results are consistent with our analysis on photon correlation given in Sec. IV(c).

V. CONCLUSION

To conclude, we have studied the transport properties of the photonic wave packet, controlled by the Rydberg interaction between two atoms in one continuum dimension. We find that the quantum statistical properties of the scattered photons can be predicted from the relative wave function of two photons as well as the second-order correlation function. With detailed calculations about the relative wave functions, we point out how to control the quantum statistical properties of the scattered photons in the confined system. To change the quantum statistical properties from bunching to antibunching or vice versa, for a certain Rydberg interaction strength between the two atoms, we adjust the detunings of frequency center of the photonic wave packet from each atom energy level spacings, respectively. On the other hand, for a cer-

tain detunings of frequency center of the photonic wave packet from each atom energy level spacings, we can regulate the Rydberg interaction strength by varying the distance between the two atoms or select two different energy levels of the Rydberg atoms. We can use the change of the quantum statistic properties to detect the detail fashion of the Rydberg interaction between the Rydberg atoms.

ACKNOWLEDGMENTS

This work is supported by National Natural Science Foundation of China under Grants No. 11121403, No. 10935010 and No. 11074261.

Appendix: Solution of Eq. (18) by the Laplace transformation

In this appendix, we present a detailed derivation of Eq. (21) by employing the Laplace transformation. Under the initial condition (19), the equation of motion (18) becomes

$$\begin{aligned} s\tilde{A}(s) &= -i\xi\tilde{A}(s) - ig\int_0^\infty dk[\tilde{B}_k(s)+\tilde{C}_k(s)], \\ s\tilde{B}_k(s) &= -i\Delta_k\tilde{B}_k(s) - ig\tilde{A}(s) - ig\int_0^\infty dp\tilde{D}_{p,k}(s), \\ s\tilde{C}_k(s) &= -i\Delta_k\tilde{C}_k(s) - ig\tilde{A}(s) - ig\int_0^\infty dp\tilde{D}_{p,k}(s), \\ s\tilde{D}_{p,q}(s) &= D_{p,q}(0) - i(\Delta_p+\Delta_q)\tilde{D}_{p,q}(s) \\ &\quad -ig[\tilde{B}_p(s)+\tilde{B}_q(s)+\tilde{C}_p(s)+\tilde{C}_q(s)]. \end{aligned} \tag{A.1}$$

By eliminating other variables, we obtain the following equation for the variable $\tilde{B}_k(s)$ as

$$\begin{aligned} &[\Delta_k - i(s+\gamma)]\tilde{B}_k(s) \\ &= 2g^2\int_{-\infty}^{\infty} d\Delta_p\left(\frac{1}{\xi - is}+\frac{1}{\Delta_p+\Delta_k-is}\right)\tilde{B}_p(s) \\ &\quad +ig\int_{-\infty}^{\infty}\frac{D_{p,k}(0)}{\Delta_p+\Delta_k-is}d\Delta_p. \end{aligned} \tag{A.2}$$

The solution of $\tilde{B}_k(s)$ can be obtained as

$$\begin{aligned} \tilde{B}_k(s) = &\frac{2\pi g G_2}{\Delta_k - i(s+\gamma)}[\widetilde{f_1}+2e^{-sl_1}\gamma(\widetilde{f_2}+\widetilde{f_3})] \\ &\times\Theta(l_1)\Theta(l_2)\Theta(l_1-l_2), \end{aligned} \tag{A.3}$$

with

$$\begin{aligned} \widetilde{f_1} &= \frac{e^{-sl_1}e^{-i(l_1-l_2)\Delta_k}}{(\Delta_k-\delta_2+i\epsilon)(\Delta_k+\delta_1-i(s+\epsilon))} + \frac{e^{-sl_2}e^{i(l_1-l_2)\Delta_k}}{(\Delta_k-\delta_1+i\epsilon)(\Delta_k+\delta_2-i(s+\epsilon))}, \\ \widetilde{f_2} &= \frac{e^{-i(l_1-l_2)\Delta_k}}{(\Delta_k+i\gamma)(\Delta_k-\delta_2+i\epsilon)(s+\epsilon+i(\Delta_k+\delta_1))} + \frac{e^{-(l_1-l_2)\gamma}}{(\gamma-\epsilon-i\delta_2)(s+\gamma+\epsilon+i\delta_1)}\left(\frac{1}{-\gamma+i\Delta_k}+\frac{1}{s+\gamma+i\xi}\right), \\ \widetilde{f_3} &= \frac{e^{-(l_1-l_2)(i\delta_2+\epsilon)}}{s+i\delta_1+i\delta_2+2\epsilon}\left(\frac{1}{(i\gamma+\delta_2-i\epsilon)(\delta_2-\Delta_k-i\epsilon)} - \frac{1}{(s+\gamma+i\delta_2+\epsilon)(s+i\delta_2+i\Delta_k+\epsilon)}\right. \\ &\quad \left. -\frac{1}{(\gamma-i\delta_2-\epsilon)(s+\gamma+i\xi)} - \frac{1}{(s+\gamma+i\delta_2+\epsilon)(s+\gamma+i\xi)}\right). \end{aligned} \tag{A.4}$$

Then we have

$$\tilde{D}_{p,q}(s) = \frac{D_{p,q}(0) - 2ig[\tilde{B}_p(s)+\tilde{B}_q(s)]}{s+i(\Delta_p+\Delta_q)}. \tag{A.5}$$

The transient solution of $D_{p,q}(t)$ can be obtained by the inverse Laplace transformation. For studying photon scattering, it is sufficient to get the long-time solution given in Eq. (21).

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